

Globally Variance-Constrained Sparse Representation for Image Set Compression

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Abstract—Sparse representation presents an efficient approach to approximately recover a signal by the linear composition of a few bases from a learnt dictionary, based on which various successful applications have been observed. However, in the scenario of data compression, its efficiency and popularity are hindered due to the extra overhead for encoding the sparse coefficients. Therefore, how to establish an accurate rate model in sparse coding and dictionary learning becomes meaningful, which has been not fully exploited in the context of sparse representation. According to the Shannon entropy inequality, the variance of data source bounds its entropy, which can reflect the actual coding bits. Hence, in this work a Globally Variance-Constrained Sparse Representation (GVCSR) model is proposed, where a variance-constrained rate model is introduced in the optimization process. Specifically, we employ the Alternating Direction Method of Multipliers (ADMM) to solve the non-convex optimization problem for sparse coding and dictionary learning, both of which have shown state-of-the-art performance in image representation. Furthermore, we investigate the potential of GVCSR in practical image set compression, where a common dictionary is trained by several key images to represent the whole image set. Experimental results have demonstrated significant performance improvements against the most popular image codecs including JPEG and JPEG2000.

Index Terms—Globally variance-constrained sparse representation, alternating direction method of multipliers, image set compression.

I. INTRODUCTION

LOSSY image compression aims to reduce redundancies among pixels while maintaining the required quality for the purpose of efficient transmission and storage. Due to the energy compaction property, transform coding approaches have shown great power in de-correlation and have been widely adopted in image compression standards, as witnessed from the Discrete Cosine Transform (DCT) in Joint Photographic Experts Group (JPEG) [1] to the Discrete Wavelet Transform (DWT) in JPEG2000 [2]. The common property of both DCT and DWT lies in that their basis functions are orthogonal to each other and meanwhile fixed despite the characteristics of the input signals. However, such inflexibility may greatly restrict their representation efficiency.

In the early 1990s, Olshausen and Field firstly proposed the sparse coding with a learnt dictionary [3] to represent an

image. Since then the sparse theory has been widely studied and advocated [4], [5]. Different from orthogonal transforms, the bases in sparse model are always over-complete and nonorthogonal. It is widely believed that the sparsity property is efficient in dealing with rich, varied and directional information contained in natural scenes [4], [6]. It has also been proven that the basis in sparse coding has the characteristics of spatially localized, oriented and bandpass, which are closely relevant to the properties of the receptive fields of simple cells. Recent studies further validated the idea that the sparse coding performs in a perceptual way that mimics the Human Visual System (HVS) on natural images [7]–[9]. Based on the sparse model, numerous applications have been successfully achieved, including image denoising [10], restoration [11]–[14], quality assessment [15]–[17], etc.

However, with respect to image compression, the whole sparse coding pipeline should be optimized in terms of both rate and distortion [18]. Despite the fact that sparse coding can provide more efficient representation than orthogonal transforms [19], its efficiency in compression tasks is however limited because encoding sparse coefficients would be rather costly. In the conventional sparse representation model, the objective function is composed of two parts, which are data fidelity term and sparsity constraint term (via ℓ_0 or ℓ_1 norm). Although it is generally acknowledged that more coefficients usually require more coding bits, the sparsity constraint term may not be able to accurately reflect the actual coding bits. Hence, this motivates researchers to improve the standard sparse representation model to accurately estimate the coding rate for the purpose of rate-distortion optimized compression. The Rate-Distortion Optimized Matching Pursuit (RDOMP) approaches [20]–[22] were proposed to address this issue, where the coding rate was estimated based on the probabilistic model of the coefficients. Despite their performance improvements on image compression, the RDOMPs may also have some limitations. Firstly, such matching pursuit based methods may suffer from the instability in obtaining the sparse coefficients [23]. Secondly, they operate and encode each sample separately, which ignores the data structure information and lacks global constraint over all input samples. This may lead to quite different representations of two similar samples and result in performance decrease [24]–[26]. Finally, the RDOMP methods are nontrivial to be incorporated with the dictionary learning algorithm and such inconsistency may also decrease the efficiency.

To overcome these problems, the Globally Variance-Constrained Sparse Representation (GVCSR) model is pro-

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TABLE I
SUMMARY OF NOTATIONS.

Notation	Meaning
Bold capital letter	A matrix.
Bold lowercase letter	A vector.
M^T	Transpose of matrix M .
$M_{i,j}$	The $(i,j)^{th}$ element of matrix M .
M_i	The i^{th} column of matrix M .
M^\dagger	Moore-Penrose pseudo-inverse of matrix M .
$\ \cdot\ _0$	Number of nonzero elements.
$\ \cdot\ _1$	$\ M\ _1 = \sum_{i,j} M_{i,j} $.
$\ \cdot\ _F$	Frobenious norm, $\ M\ _F = \sqrt{\sum_{i,j} M_{i,j}^2}$.
$\ \cdot\ _2$	Vector Euclidean norm, $\ v\ _2 = \sqrt{\sum_i v_i^2}$.
$tr(\cdot)$	Sum of the diagonal entries of a matrix.
$\langle A, B \rangle$	Inner product of matrix, $\langle A, B \rangle = tr(A^T B)$.

posed in this work, where a variance-constraint term is introduced into the objective function in sparse representation. By incorporating the rate model, minimization of the objective function turns out to be a joint rate-distortion optimization problem, which can be efficiently solved by Alternating Direction Method of Multipliers (ADMM) [27] in both sparse coding and dictionary learning. In particular, here “globally” aims to emphasize the way for solving the model, which significantly distinguishes from the separate manner in matching pursuit. Therefore, such optimization based method can effectively utilize the intrinsic data structure and reduce the instability.

The major contributions of this work are as follows.

- We propose a novel sparse representation model, by which the rate-distortion jointly optimized sparse coding and dictionary learning can be coherently achieved.
- We solve the non-convex optimization problem with the ADMM in a scientifically sound way. In such way, the sparse coefficient correlation between similar image patches can be implicitly taken into account during optimization, which distinguishes it from the patch-wise updating in MP methods.
- We demonstrate effectiveness and potential of the proposed method in practical image set compression, where the dictionary is trained from a few key images to represent the whole image set. Experimental results have demonstrated impressive coding gains against the state-of-the-art sparse coding and dictionary learning algorithms as well as the popular image codecs.

For brevity, we summarize some frequently used notations in Table I. The rest of the paper is organized as follows. The related works are briefly summarized in Section II. In Section III, we present the GVCSSR model and the solution to solve it via ADMM. Section IV evaluates the efficiency of GVCSSR and further demonstrates its power in image set compression. Finally, we conclude this paper in Section V.

II. RELATED WORKS

Sparse theory claims that signals can be well recovered by a few bases from an over-complete dictionary, where the dictionary is assumed to be highly adaptive to a set of signals within a limited subspace. The two basic problems in sparse representation are dictionary learning and sparse decomposition (or sparse coding). In particular, the objective function of dictionary learning can be formulated as follows,

$$(\mathbf{D}, \{\mathbf{A}_i\}) = \arg \min_{\mathbf{D}, \{\mathbf{A}_i\}} \sum_i \|\mathbf{T}_i - \mathbf{D}\mathbf{A}_i\|_2^2, \text{ s.t. } \|\mathbf{A}_i\|_0 \leq L, \\ \|\mathbf{D}_j\|_2^2 \leq 1, \forall j \in \{1, 2, \dots, M\} \quad (1)$$

where $\mathbf{D} \in \mathbb{R}^{N \times M}$ is the redundant dictionary with M bases. $\mathbf{T}_i \in \mathbb{R}^{N \times 1}$ indicates the training data, and $\mathbf{A}_i \in \mathbb{R}^{M \times 1}$ is the corresponding sparse representation vector, whose ℓ_0 norm is constrained by a given sparse level L . This is a non-convex optimization problem which is difficult to solve due to the ℓ_0 norm. Typical algorithms for dictionary learning include the Method of Optimal Directions (MOD) [28], [29], KSVD [30], Online Dictionary Learning (ODL) [31] and Recursive Least Square (RLS) [32].

With respect to the trained dictionary \mathbf{D} , the sparse decomposition calculates appropriate coefficients \mathbf{A}_i for the input signal \mathbf{S}_i ,

$$\mathbf{A}_i = \arg \min_{\mathbf{A}_i} \|\mathbf{S}_i - \mathbf{D}\mathbf{A}_i\|_2^2, \text{ s.t. } \|\mathbf{A}_i\|_0 \leq L, \quad (2)$$

which is a subproblem of (1). Several suboptimal solutions have been proposed to solve the problem, including ℓ_1 convex relaxation approach [33] and the well-known Matching Pursuit Family (MPF) algorithms that operate in an iteratively greedy way [34].

Towards data compression, extensive efforts have been made to improve the typical sparse model. On one hand, it is known that the dictionary is critical to the coding performance, which is required to be adaptive to the image content. On the other hand, it would be rather expensive to represent an adaptive dictionary. Therefore, the authors in [35] presented a double sparsity framework, in which both the dictionary and coefficients were assumed to be sparse and easy to code. Based on that, in [36] the adaptively learnt dictionary was encoded with the coefficients in an attempt to achieve both efficiency and adaptivity. Furthermore, in [37] an online dictionary learning algorithm for intra-frame video coding was proposed, where the dictionary was dynamically updated across video frames and only the dictionary changes were encoded in the stream. However, the coding efficiency in those approaches was highly limited due to the considerable bitrate consumptions of the dictionary.

Therefore, in most of the existing works, a global dictionary was offline trained and shared in both encoder and decoder. Typical works attempted to train a content-aware dictionary for facial image compression [38]–[40] and screen image compression [41]. For general image compression, several approaches have been proposed to improve the representation ability of the global dictionary. In [42], the RLS dictionary learning algorithm [32] was employed in the 9/7 wavelet

domain. Recently, a new sparse dictionary learning model was proposed by imposing a compressibility constraint on the coefficients [23]. However, there is still much room to improve the compression efficiency of sparse model. In conventional sparse models, the sparsity constraint term cannot well reflect the actual coding bits of the coefficients. This motivates us to propose the GVCSR model for the purpose of rate-distortion jointly optimized sparse representation.

III. GLOBALLY VARIANCE-CONSTRAINED SPARSE REPRESENTATION

In this section, we firstly present the Globally Variance-Constrained Sparse Representation (GVCSR) model. The effective solutions for sparse coding and dictionary learning are subsequently introduced. Finally, some implementation issues are provided.

A. Rate-Distortion Optimized Sparse Representation

For data compression, the objective function that takes the coding rate into consideration is formulated as follows,

$$\begin{aligned} \arg \min_{\mathbf{A}, \mathbf{D}} \left\{ \frac{1}{2} \|\mathbf{S} - \mathbf{DA}\|_F^2 + \lambda \cdot r(\mathbf{A}) \right\}, \\ \text{s.t. } \|\mathbf{A}_i\|_0 \leq L, \forall i \in \{1, \dots, K\}, \\ \|\mathbf{D}_j\|_2^2 \leq 1, \forall j \in \{1, 2, \dots, M\}, \end{aligned} \quad (3)$$

where $\mathbf{S} \in \mathbb{R}^{N \times K}$ is the stack of input vectors \mathbf{S}_i , $\mathbf{A} \in \mathbb{R}^{M \times K}$ is the stack of the corresponding sparse coefficient vectors \mathbf{A}_i , K denotes the number of the input samples. $r(\cdot)$ indicates a function that can represent the coding rate of the coefficients. This formula can be well expressed by the rate-distortion optimization in common image/video compressions [18], where λ controls the relative importance between the rate and distortion. ℓ_0 norm of the coefficients is still critical in order to obtain a sparse approximation. Subsequently, the problem turns to how to accurately estimate the coding rate and achieve such optimization in sparse coding and dictionary learning.

Based on the *Shannon's* information theory, the entropy of a data source indicates the average number of bits required to represent it. However, it is difficult to estimate the probability density function of coefficients and formulate the entropy minimization problem. Fortunately, the entropy can be bounded by the variance of data according to the *Shannon* entropy inequality [43],

$$H(\mathbf{A}) \leq \log \left(\sqrt{2\pi e V(\mathbf{A})} \right), \quad (4)$$

where $H(\mathbf{A})$ and $V(\mathbf{A})$ indicate the entropy and the variance of coefficients, respectively. Note that the inequality is tight as the equality holds for Gaussian distributions. Another benefit is that the variance can be feasibly estimated by,

$$V(\mathbf{A}) = \text{tr}(\mathbf{AZA}^T), \quad (5)$$

where

$$\mathbf{Z} = \begin{pmatrix} K-1 & \dots & -1 \\ \vdots & \ddots & \vdots \\ -1 & \dots & K-1 \end{pmatrix} \in \mathbb{R}^{K \times K}. \quad (6)$$

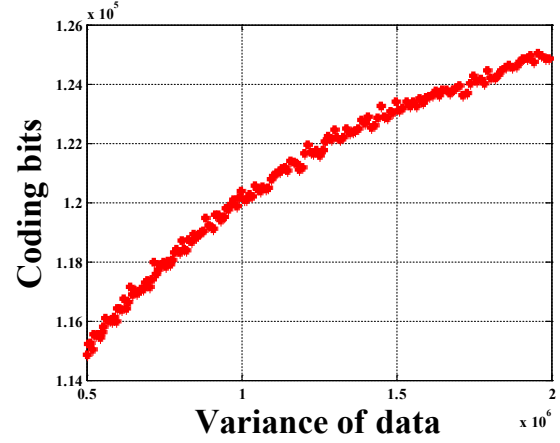


Fig. 1. Relationship between the data variance and the corresponding coding bits.

The diagonal elements in \mathbf{Z} equal $K-1$ and others equal -1 .

Due to the *Shannon* entropy inequality and the feasibility of variance estimation, we propose to minimize the variance instead of entropy. The reason lies in that minimizing variance encourages the coefficients to be close to each other, which can be more friendly to compression. To further validate this, we randomly generate data sources and encode them by Huffman coding. The relationship between the variances of the input data and the corresponding coding bits is plotted in Fig. 1, from which one can discern that the variance exhibits a nearly perfect linear relationship to the actual bitrate. The clear relationship helps us establish the rate model in sparse representation.

Therefore, the objective function in (3) can be formulated as follows,

$$\begin{aligned} \arg \min_{\mathbf{A}, \mathbf{D}} \left\{ \frac{1}{2} \|\mathbf{S} - \mathbf{DA}\|_F^2 + \frac{\beta}{2} \text{tr}(\mathbf{AZA}^T) \right\}, \\ \text{s.t. } \|\mathbf{A}_i\|_0 \leq L, \forall i \in \{1, \dots, K\}, \\ \|\mathbf{D}_j\|_2^2 \leq 1, \forall j \in \{1, 2, \dots, M\}, \end{aligned} \quad (7)$$

where $\beta/2$ is introduced for computational convenience. Generally speaking, the ℓ_0 norm constraint can be approximately solved by Lagrangian method,

$$\begin{aligned} \arg \min_{\mathbf{A}, \mathbf{D}} \left\{ \frac{1}{2} \|\mathbf{S} - \mathbf{DA}\|_F^2 + \alpha \|\mathbf{A}\|_0 + \frac{\beta}{2} \text{tr}(\mathbf{AZA}^T) \right\}, \\ \text{s.t. } \|\mathbf{D}_i\|_2^2 \leq 1, \forall i \in \{1, 2, \dots, M\}. \end{aligned} \quad (8)$$

To solve the non-convex optimization problem effectively, a practical relaxation is to split the problem into two separable parts and update \mathbf{A} and \mathbf{D} alternately, i.e. the GVCSR based sparse coding and GVCSR based dictionary learning.

B. GVCSR based Sparse Coding

The GVCSR based sparse coding given the dictionary \mathbf{D} is a subproblem of (8), which can be formulated as follows,

$$\mathbf{A} = \arg \min_{\mathbf{A}} \left\{ \frac{1}{2} \|\mathbf{S} - \mathbf{DA}\|_F^2 + \alpha \|\mathbf{A}\|_0 + \frac{\beta}{2} \text{tr}(\mathbf{AZA}^T) \right\}. \quad (9)$$

To solve this, the Alternating Direction Method of Multipliers (ADMM) [27] is employed in this work. The ADMM was originally introduced in the early 1970s [44], and has been widely used in machine learning, computer vision and signal processing [27], [45]–[47]. Its convergence properties for both convex and non-convex problems were also intensively addressed [27], [48]. It is known that the ADMM is efficient in dealing with the following problem,

$$\arg \min_{\mathbf{x}, \mathbf{y}} \{f(\mathbf{x}) + g(\mathbf{y})\}, \text{ s.t. } \mathbf{B}(\mathbf{x}) + \mathbf{C}(\mathbf{y}) - \mathbf{d} = 0, \quad (10)$$

where \mathbf{x} , \mathbf{y} and \mathbf{d} could be either vectors or matrices. \mathbf{B} and \mathbf{C} are linear operators that define the constraint function. By introducing Lagrangian multiplier vector \mathbf{R} , the augmented Lagrangian function can be formed as follows,

$$\begin{aligned} \zeta_\mu(\mathbf{x}, \mathbf{y}, \mathbf{R}) = & f(\mathbf{x}) + g(\mathbf{y}) + \langle \mathbf{B}(\mathbf{x}) + \mathbf{C}(\mathbf{y}) - \mathbf{d}, \mathbf{R} \rangle \\ & + \frac{\mu}{2} \|\mathbf{B}(\mathbf{x}) + \mathbf{C}(\mathbf{y}) - \mathbf{d}\|_2^2, \end{aligned} \quad (11)$$

where $\mu > 0$ is the penalty parameter. ADMM updates the estimate of $\mathbf{x}, \mathbf{y}, \mathbf{R}$ via solving the following problems alternately,

$$\begin{cases} \mathbf{x}^{k+1} = \arg \min_{\mathbf{x}} \zeta_\mu(\mathbf{x}, \mathbf{y}^k, \mathbf{R}^k) \\ \mathbf{y}^{k+1} = \arg \min_{\mathbf{y}} \zeta_\mu(\mathbf{x}^{k+1}, \mathbf{y}, \mathbf{R}^k) \\ \mathbf{R}^{k+1} = \mathbf{R}^k + \mu (\mathbf{B}(\mathbf{x}^{k+1}) + \mathbf{C}(\mathbf{y}^{k+1}) - \mathbf{d}). \end{cases} \quad (12)$$

To solve (9), we adopt the ADMM [27] with some delicate derivations. Firstly, two auxiliary variables \mathbf{J} and \mathbf{G} are introduced in order to fit the form in (10),

$$\begin{aligned} \arg \min_{\mathbf{A}, \mathbf{J}, \mathbf{G}} \left\{ \frac{1}{2} \|\mathbf{S} - \mathbf{D}\mathbf{J}\|_F^2 + \alpha \|\mathbf{A}\|_0 + \frac{\beta}{2} \text{tr}(\mathbf{G}\mathbf{Z}\mathbf{G}^T) \right\}, \\ \text{s.t. } \mathbf{A} = \mathbf{J}, \mathbf{A} = \mathbf{G}, \end{aligned} \quad (13)$$

by setting the parameters in (10) as follows

$$\begin{cases} \mathbf{x} = \mathbf{A} \\ \mathbf{y} = \begin{bmatrix} \mathbf{J} \\ \mathbf{G} \end{bmatrix} \\ f(\mathbf{x}) = \alpha \|\mathbf{A}\|_0 \\ g(\mathbf{y}) = \frac{1}{2} \|\mathbf{S} - \mathbf{D}\mathbf{J}\|_F^2 + \frac{\beta}{2} \text{tr}(\mathbf{G}\mathbf{Z}\mathbf{G}^T) \\ \mathbf{B}(\mathbf{x}) = \begin{bmatrix} \mathbf{A} \\ \mathbf{A} \end{bmatrix}, \mathbf{C}(\mathbf{y}) = \begin{bmatrix} -\mathbf{J} \\ -\mathbf{G} \end{bmatrix}, \mathbf{d} = \mathbf{0}, \end{cases} \quad (14)$$

Then the augmented Lagrangian function of (13) can be formulated by,

$$\begin{aligned} \zeta_\mu(\mathbf{A}, \mathbf{J}, \mathbf{G}, \mathbf{R}) = & \frac{1}{2} \|\mathbf{S} - \mathbf{D}\mathbf{J}\|_F^2 + \alpha \|\mathbf{A}\|_0 + \frac{\beta}{2} \text{tr}(\mathbf{G}\mathbf{Z}\mathbf{G}^T) \\ & + \langle \mathbf{A} - \mathbf{J}, \mathbf{R}_0 \rangle + \frac{\mu}{2} \|\mathbf{A} - \mathbf{J}\|_F^2 \\ & + \langle \mathbf{A} - \mathbf{G}, \mathbf{R}_1 \rangle + \frac{\mu}{2} \|\mathbf{A} - \mathbf{G}\|_F^2, \end{aligned} \quad (15)$$

where $\mathbf{R} \triangleq [\mathbf{R}_0; \mathbf{R}_1]$ is the Lagrange multiplier matrix.

The variables \mathbf{A} , \mathbf{J} and \mathbf{G} can be alternately updated by minimizing the augmented Lagrangian function ζ with other variables fixed. In this model, each variable can be updated

with a closed form solution. Regarding \mathbf{A} , it can be updated as follows,

$$\begin{aligned} \mathbf{A} = & \arg \min_{\mathbf{A}} \left\{ \alpha \|\mathbf{A}\|_0 + \langle \mathbf{A} - \mathbf{J}, \mathbf{R}_0 \rangle + \frac{\mu}{2} \|\mathbf{A} - \mathbf{J}\|_F^2 \right. \\ & \left. + \langle \mathbf{A} - \mathbf{G}, \mathbf{R}_1 \rangle + \frac{\mu}{2} \|\mathbf{A} - \mathbf{G}\|_F^2 \right\} \\ = & \arg \min_{\mathbf{A}} \left\{ \alpha \|\mathbf{A}\|_0 + \frac{\mu}{2} \left\| \mathbf{A} - \mathbf{J} + \frac{\mathbf{R}_0}{\mu} \right\|_F^2 \right. \\ & \left. + \frac{\mu}{2} \left\| \mathbf{A} - \mathbf{G} + \frac{\mathbf{R}_1}{\mu} \right\|_F^2 \right\} \\ = & H_{\sqrt{\alpha/\mu}} \left\{ \frac{1}{2} \left(\mathbf{J} + \mathbf{G} - \frac{\mathbf{R}_0 + \mathbf{R}_1}{\mu} \right) \right\}, \end{aligned} \quad (16)$$

where

$$\mathbf{H}_\varepsilon(\mathbf{X}) \triangleq \begin{pmatrix} h_\varepsilon(\mathbf{X}_{11}) & \cdots & h_\varepsilon(\mathbf{X}_{1n}) \\ \vdots & \ddots & \vdots \\ h_\varepsilon(\mathbf{X}_{m1}) & \cdots & h_\varepsilon(\mathbf{X}_{mn}) \end{pmatrix}. \quad (17)$$

and

$$h_\varepsilon(x) \triangleq \begin{cases} x, & \text{if } |x| > \varepsilon \\ 0, & \text{if } |x| \leq \varepsilon \end{cases} \quad (18)$$

is a hard threshold operator. With respect to \mathbf{J} and \mathbf{G} , we can update them as follows,

$$\begin{aligned} \mathbf{J} = & \arg \min_{\mathbf{J}} \left\{ \frac{1}{2} \|\mathbf{S} - \mathbf{D}\mathbf{J}\|_F^2 + \langle \mathbf{A} - \mathbf{J}, \mathbf{R}_0 \rangle + \frac{\mu}{2} \|\mathbf{A} - \mathbf{J}\|_F^2 \right\} \\ = & \arg \min_{\mathbf{J}} \left\{ \frac{1}{2} \|\mathbf{S} - \mathbf{D}\mathbf{J}\|_F^2 + \frac{\mu}{2} \left\| \mathbf{A} - \mathbf{J} + \frac{\mathbf{R}_0}{\mu} \right\|_F^2 \right\} \\ = & \mathbf{V}_D (\Sigma_D^T \Sigma_D + \mu \mathbf{I})^{-1} \mathbf{V}_D^T (\mathbf{D}^T \mathbf{S} + \mu \mathbf{A} + \mathbf{R}_0), \quad (19) \\ \mathbf{G} = & \arg \min_{\mathbf{G}} \left\{ \frac{\beta}{2} \text{tr}(\mathbf{G}\mathbf{Z}\mathbf{G}^T) + \langle \mathbf{A} - \mathbf{G}, \mathbf{R}_1 \rangle + \frac{\mu}{2} \|\mathbf{A} - \mathbf{G}\|_F^2 \right\} \\ = & \arg \min_{\mathbf{G}} \left\{ \frac{\beta}{2} \text{tr}(\mathbf{G}\mathbf{Z}\mathbf{G}^T) + \frac{\mu}{2} \left\| \mathbf{A} - \mathbf{G} + \frac{\mathbf{R}_1}{\mu} \right\|_F^2 \right\} \\ = & \beta (\mu \mathbf{A} + \mathbf{R}_1) \mathbf{V}_Z (\Sigma_Z + \mu \mathbf{I})^{-1} \mathbf{V}_Z^T, \end{aligned} \quad (20)$$

where $\mathbf{U}_D \Sigma_D \mathbf{V}_D^T$ and $\mathbf{U}_Z \Sigma_Z \mathbf{V}_Z^T$ are the full Singular Value Decomposition (SVD) of \mathbf{D} and \mathbf{Z} , respectively. Finally, the Lagrangian multiplier \mathbf{R}_0 and \mathbf{R}_1 are updated,

$$\mathbf{R}_0^{j+1} = \mathbf{R}_0^j + \mu^j (\mathbf{A}^{j+1} - \mathbf{J}^{j+1}), \quad (21)$$

$$\mathbf{R}_1^{j+1} = \mathbf{R}_1^j + \mu^j (\mathbf{A}^{j+1} - \mathbf{G}^{j+1}), \quad (22)$$

where j indicates the iteration times.

In previous ADMM approach [27], the penalty parameter μ is fixed. To accelerate the convergence, an updating strategy for the penalty parameter is proposed in [49], which can be formulated as follows,

$$\mu^{j+1} = \min(\rho \mu^j, \mu_{max}), \quad (23)$$

where μ_{max} is an upper bound of the penalty term. $\rho \geq 1$ is a constant.

The optimization process of the GVCSR based sparse coding performs iteratively and stops until convergence. After that, the globally variance-constrained sparse coding can be achieved. The detailed procedure is presented in Algorithm 1.

Algorithm 1: GVCSR based sparse coding in (9).

Input:

- Input data S
- Dictionary D
- Parameters $\alpha > 0$ and $\beta > 0$

Output:

- Sparse representation coefficients A

```

1 Initialization:
2 Set  $A^0 = J^0 = G^0 = D^\dagger S, \mu^0 = 1e - 2, \mu_{max} =$ 
    $1e8, \rho = 1.2, R_0 = R_1 = 0, \epsilon = 1e - 5,$  and  $j = 0;$ 
3 while not convergence do
4   Fix  $J^j$  and  $G^j$  to update  $A^{j+1}$  by (16);
5   Fix  $A^{j+1}$  and  $G^j$  to update  $J^{j+1}$  by (19);
6   Fix  $A^{j+1}$  and  $J^{j+1}$  to update  $G^{j+1}$  by (20);
7   Update Lagrange multipliers:
      $R_0^{j+1} = R_0^j + \mu^j (A^{j+1} - J^{j+1}),$ 
      $R_1^{j+1} = R_1^j + \mu^j (A^{j+1} - G^{j+1});$ 
8   Update penalty parameter  $\mu$ :
      $\mu^{j+1} = \min(\rho\mu^j, \mu_{max});$ 
9    $j \leftarrow j + 1;$ 
10  Check convergence: if  $\|A^j - J^j\|/\|A^j\| \leq \epsilon$  and
      $\|A^j - G^j\|/\|A^j\| \leq \epsilon$  and  $\|A^j - A^{j-1}\|/\|A^j\| \leq \epsilon,$ 
     then stop;
11 end
```

C. GVCSR based Dictionary Learning

In order to make the dictionary learning consistent with the proposed sparse coding strategy, we further solve the dictionary updating problem based on the overall GVCSR model in (8). The detailed procedure of the proposed algorithm is described in Algorithm 2. The updating process of sparse coefficients A is the same as that in Algorithm 1. After the convergence of A , the dictionary D can be updated with the optimized A as follows,

$$D = \arg \min_D \left\{ \frac{1}{2} \|S - DA\|_F^2 \right\} \quad (24)$$

s.t. $\|D_i\|_2 \leq 1, \forall i \in \{1, 2, \dots, M\}.$

This can be solved via performing SVD on the residuals [30]. Then, the parameters A, J, G should be further updated according to the new dictionary, while the Lagrangian multipliers R_0 and R_1 are reset to zero vectors. Moreover, the penalty parameter μ is updated as follows,

$$\mu^{j+1} = \frac{\kappa \cdot \alpha}{\min |A_{nz}^j|}, \quad (25)$$

where A_{nz}^j denotes all nonzero elements in A^j . $\kappa \geq 1$ is a parameter that guides how largely the $\|A\|_0$ would change in the next iteration when updating A . κ is empirically set as 4 in this work.

D. Implementation Issues

In (19) & (20), the full-SVD should be performed when updating the variables. The computational complexity is $O(N^3)$, which is inappropriate for large matrix processing. Practically,

Algorithm 2: GVCSR based dictionary learning in (8).

Input:

- Input data S
- Parameters $\alpha > 0$ and $\beta > 0$

Output:

- Trained dictionary D
- Sparse representation coefficients A

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1 Initialization:
2 Initialize  $D;$ 
3 Set  $A^0 = J^0 = G^0 = D^\dagger S, \mu^0 = 1e - 2, \mu_{max} =$ 
    $1e8, \rho = 1.2, R_0^0 = R_1^0 = 0, \epsilon = 1e - 5,$  and  $j = 0;$ 
4 while not convergence do
5   Updating Sparse Coefficients:
6   Fix  $J^j$  and  $G^j$  to update  $A^{j+1}$  by (16);
7   Fix  $A^{j+1}$  and  $G^j$  to update  $J^{j+1}$  by (19);
8   Fix  $A^{j+1}$  and  $J^{j+1}$  to update  $G^{j+1}$  by (20);
9   Update Lagrange multipliers:
      $R_0^{j+1} = R_0^j + \mu^j (A^{j+1} - J^{j+1}),$ 
      $R_1^{j+1} = R_1^j + \mu^j (A^{j+1} - G^{j+1});$ 
10  Update penalty parameter  $\mu$ :
      $\mu^{j+1} = \min(\rho\mu^j, \mu_{max});$ 
11   $j \leftarrow j + 1;$ 
12  Check convergence:
13  if  $\|A^j - J^j\|/\|A^j\| \leq \epsilon$  and  $\|A^j - G^j\|/\|A^j\| \leq \epsilon$ 
     and  $\|A^j - A^{j-1}\|/\|A^j\| \leq \epsilon$  then
14    Updating Dictionary:
15    Updating  $D$  by solving (24);
16     $A^{j+1} = J^{j+1} = G^{j+1} = D^\dagger S;$ 
17     $R_0^{j+1} = R_1^{j+1} = 0;$ 
18    Update  $\mu^{j+1}$  by (25);
19     $j \leftarrow j + 1;$ 
20  end
21 end
```

the dimension of matrix $Z \in \mathbb{R}^{K \times K}$ can be extremely high due to the large number of image blocks. Therefore, we propose a fast full-SVD algorithm for the specific matrix Z .

Let $V_Z = \{v_z^1, \dots, v_z^K\}$ be the singular vectors of Z , and the corresponding singular values are denoted as $\{\sigma_z^1, \dots, \sigma_z^K\}$. It is easy to derive that Z has only two singular values, 0 and K respectively. The singular value 0 corresponds to an all-one singular vector, and the singular value K corresponds to $K - 1$ singular vectors. Thus we have

$$\Sigma_Z = \begin{pmatrix} 0 & & & \\ & K & & \\ & & \ddots & \\ & & & K \end{pmatrix} \in \mathbb{R}^{K \times K}. \quad (26)$$

Based on the principle of the SVD, the corresponding singular vectors must satisfy,

$$\langle v_z^i, v_z^j \rangle = 0 \text{ and } \langle v_z^i, (1, \dots, 1)^T \rangle = 0, \quad (27)$$

$\forall i \neq j \in \{2, \dots, K\},$

and $v_z^1 = (1, \dots, 1)^T$ is the all-one vector. Equivalently, the problem can be reformulated to find an orthogonal matrix V_Z

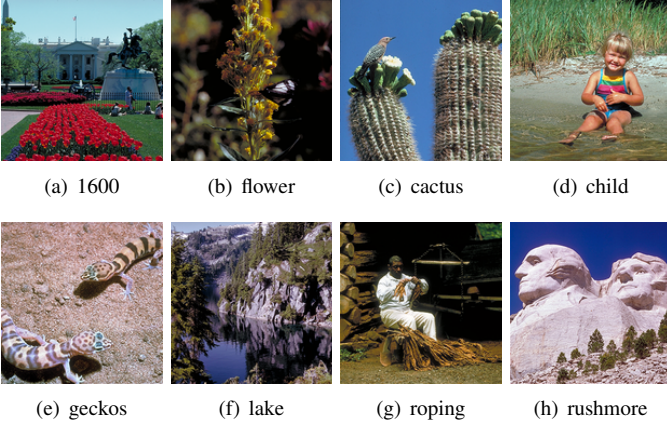


Fig. 2. Illustration of the test images from CSIQ database [50].

given v_z^1 . Consequently, we can construct the singular vector matrix as follows,

$$V_Z = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & \vdots & 1 & 1 \\ \vdots & 0 & -2 & 1 & \vdots & 1 \\ 1 & \vdots & 0 & \ddots & 1 & \vdots \\ 1 & 0 & \vdots & 0 & 2-K & 1 \\ 1 & 0 & 0 & 0 & 0 & 1-K \end{pmatrix} \in \mathbb{R}^{K \times K}, \quad (28)$$

followed by a normalization process to satisfy $\|v_z^i\|_2^2 = 1$ for $i \in \{1, \dots, K\}$. Compared with the normal full-SVD decomposition, such straightforward constructing method is very fast as its complexity is reduced to $O(N)$.

According to the convergence theory of the ADMM [48], the following conclusion can be achieved,

Theorem 1. *If the penalty parameter μ is chosen to be larger than $\{\lambda_{\max}(\mathbf{D}^T \mathbf{D})\}^2$ (where $\lambda_{\max}(\bullet)$ denotes the largest eigenvalue) and the sequence $\{(\mathbf{J}^k, \mathbf{A}^k, \mathbf{G}^k)\}$ generated from the ADMM has a cluster point $\{(\mathbf{J}^*, \mathbf{A}^*, \mathbf{G}^*)\}$, then \mathbf{A}^* is a critical point of (13).*

IV. EXPERIMENTAL RESULTS

In this section, the performance of the proposed GVCSR model is validated from two perspectives. Firstly, we compare the proposed method with other sparse coding and dictionary learning algorithms in image representation. Subsequently, we apply it to the practical image set compression application.¹

A. GVCSR for Image Representation

In these experiments, natural images from the public CSIQ dataset [50] are utilized for evaluation, and some examples are shown in Fig. 2. Each image is partitioned into 8×8 non-overlapped blocks for training their dictionary.

Firstly, the changing tendencies of distortion, rate and the overall objective function during the ADMM iterations are

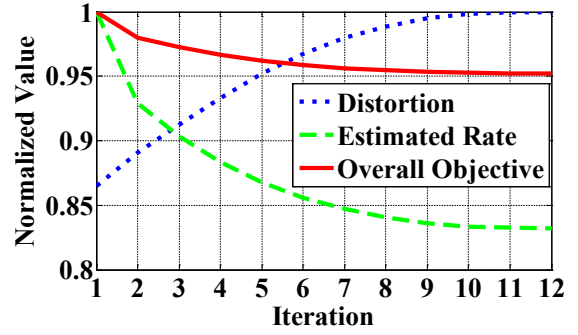


Fig. 3. Variations of distortion, estimated rate and the overall objective function value during the ADMM iterations, which are represented by blue dotted line, green dashed line and red solid line, respectively. The horizontal axis denotes the iteration times.

shown in Fig. 3. It is worth mentioning that the rate is estimated by the variance of sparse coefficients as described in III-A, and the overall objective value is calculated by (8). All the values on the three curves are normalized to the same interval for viewing convenience. From this figure we can observe that the proposed method can achieve a better tradeoff in terms of the overall objective function by greatly decreasing the coding rate while keeping the distortions slightly increased. As a result, the coding performance can be improved via the proposed algorithm.

Subsequently, we compare the GVCSR based sparse coding algorithm described in Section III-B with other sparse coding approaches. Specifically, three popular sparse decomposition methods are used for comparison, and all of them can be categorized into the separately updated matching pursuit algorithms. The first one is the standard OMP algorithm [51], where the iteration process stops until the ℓ_0 norm of coefficients reaches the limited value L . The second one is similar but the stop criterion is determined by the error energy,

$$\mathbf{A}_i = \arg \min_{\mathbf{A}_i} \|\mathbf{A}_i\|_0, \text{ s.t. } \|\mathbf{S}_i - \mathbf{D}\mathbf{A}_i\|_2^2 < \epsilon. \quad (29)$$

Those two methods are denoted as OMP_L and OMP_E , respectively. Other than these two, the RDOMP method [20]–[22] is based on the probability distribution of coefficients.

In Fig. 4, the rate-distortion comparisons are illustrated in terms of different values of completeness, where the completeness is defined as $\gamma \triangleq \frac{M}{N}$ considering the dictionary $\mathbf{D} \in \mathbb{R}^{N \times M}$. It should be noted that the rate is obtained by concrete coefficient encoding, where the value and position of nonzero coefficients are both encoded by run-level method. It can be observed that the rate-distortion performance can be significantly improved by the proposed algorithm. This may benefit from the global optimization of the proposed method that jointly considers the distortion and coding rate. Furthermore, from the figure one can also discern that the improvements are more obvious for larger γ . Obviously the sparse coefficients would become more sparse for larger γ . Thereby the matching pursuit based methods would suffer from their potential instability due to the increasing independency among coefficients, while the proposed method can effectively solve this by global optimization to significantly

¹The source code of this paper will be published if the paper is accepted.

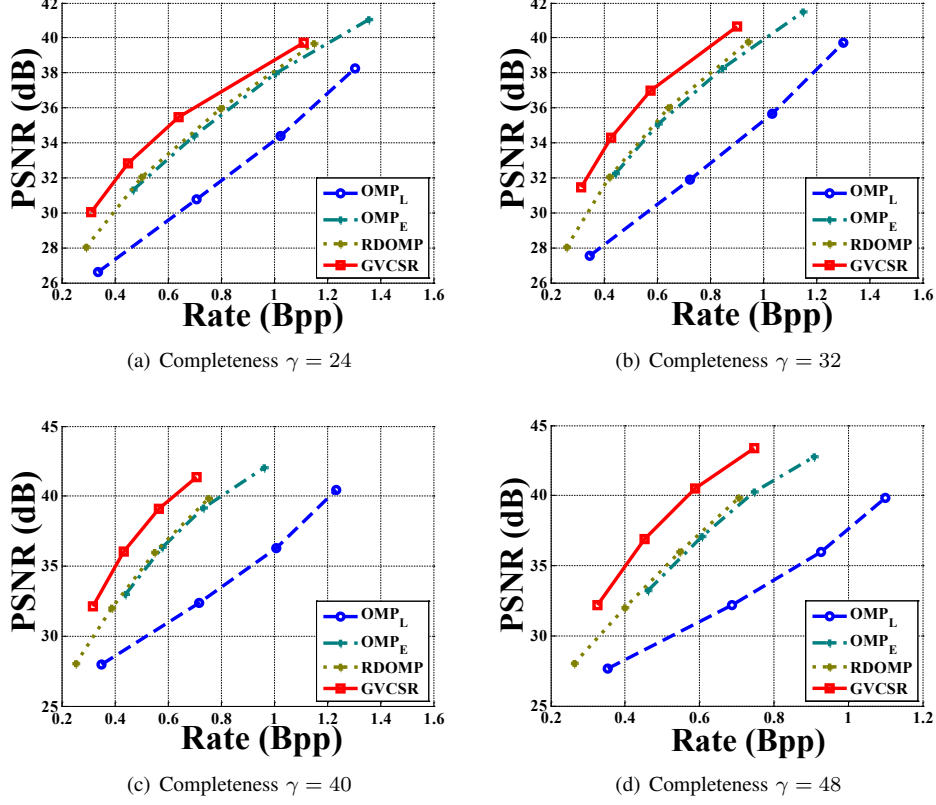


Fig. 4. Rate-distortion performance comparisons with three matching pursuit based algorithms, in terms of dictionaries with different completeness values.

TABLE II
RUNNING TIME COMPARISON BETWEEN GVCSR AND OTHER
DICTIONARY LEARNING ALGORITHMS. (TESTED ON WINDOWS PC WITH
AN INTER CORE CPU I7-4770 @ 3.40 GHz). THE TIME IS MEASURED BY
SECOND PER ITERATION.

	KSVD	MOD	ODL	RLS	GVCSR
Time (sec)	2.23	0.59	1.88	5.16	2.72

reduce the coding bits.

Finally, we evaluate the performance of the GVCSR based dictionary learning scheme described in Section III-C by comparing it with the state-of-the-art algorithms, including MOD [28], KSVD [30], ODL [31] and RLS [32]. For fair comparison, the initial dictionary in each algorithm is the same, which is randomly selected from the training set. The maximum iteration time is also set to be identical, and the parameters in each scheme are set based on their recommendation. The GVCSR based sparse coding method is employed for generating the sparse coefficients. The averaged results of actual coding rate and Peak Signal to Noise Ratio (PSNR) on the CSIQ dataset are shown in Fig. 5, from which we can see that the proposed scheme can achieve impressive improvements comparing with others in terms of rate-distortion performance. We also compare the time complexity of the proposed method with other schemes in Table II, from which the runtime of GVCSR is close to the KSVD and could be further improved by using parallel and GPU optimization.

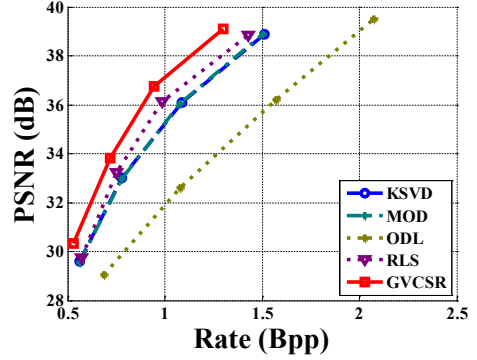


Fig. 5. Performance comparisons with the state-of-the-art dictionary learning algorithms, including MOD [28], KSVD [30], ODL [31] and RLS [32].

B. Image Set Compression based on GVCSR Model

In this subsection, we propose to apply the proposed algorithm in image set compression. Generally speaking, an image set contains similar objects of interest captured from different luminance conditions and viewpoints, indicating strong correlations among images. However, traditional single image compression methods only explore intra-image dependencies while the inter-image dependencies have been ignored.

Fortunately, it is interesting to find that the proposed GVCSR model can effectively solve these problems, where the dictionary can be learnt by a subset of images to avoid the dependency problem and meanwhile the inter-image correla-

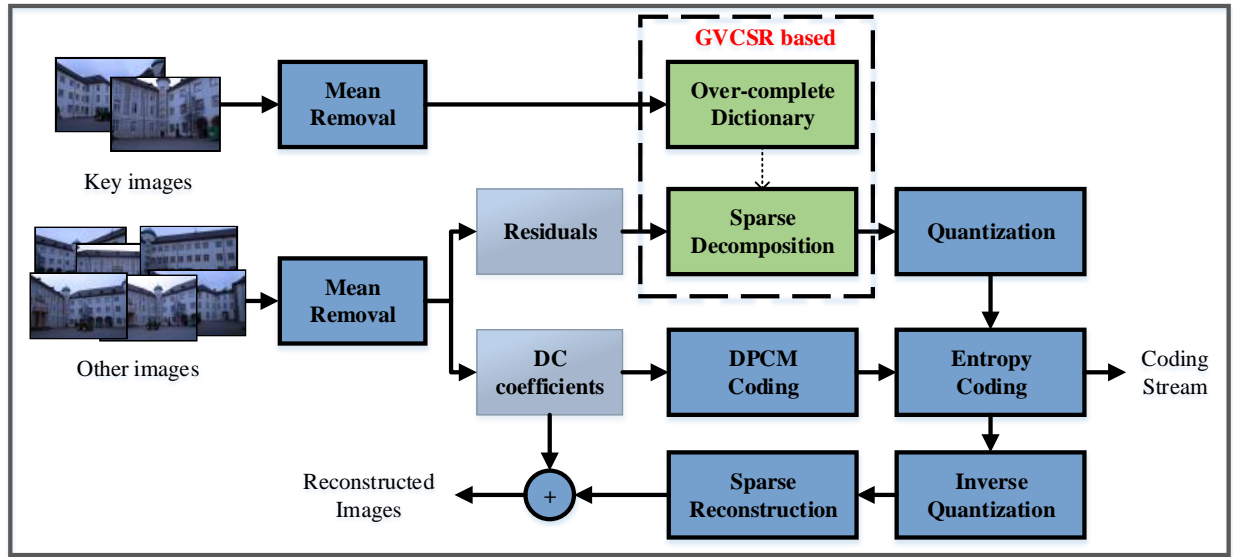


Fig. 6. Image set compression framework based on the GVCSR model. The dictionary is trained from a few key images from the image set by applying GVCSR based dictionary learning and the GVCSR based sparse coding in terms of the learnt dictionary is applied for coding the other images.

tions can also be implicitly utilized. The overall framework for image set compression is shown in Fig. 6. Specifically, the GVCSR based dictionary learning is used to train a common dictionary using a few key images from the image set. For the remaining images, the GVCSR based sparse coding with the learnt dictionary is applied to efficiently represent them. Images are partitioned into 8×8 non-overlapped blocks for training and compression. The compression scheme is composed of two parts. Firstly, the Direct-Current (DC) component, i.e. the mean value, of each image patch is the most significant band and is encoded by Differential Pulse Code Modulation (DPCM) coding, where the DC value of current block can be predicted from its left, top, top-left and top-right blocks. Secondly, the residuals after removing mean value are processed by sparse decomposition, and the sparse coefficients are then scalar-quantized and compressed by run-level method with Huffman coding. Regarding the decoder side, the sparse reconstruction is performed using the decoded sparse coefficients to obtain the distorted residuals, and the reconstructed images can be finally achieved by adding the DC components.

In the following experiments, one third of the images in an image set are used for dictionary training and the others are compressed for testing. The dictionary is trained from 8×8 image patches, and its completeness value is set to be $\gamma = 16$, indicating the dictionary has $8 \times 8 \times 16 = 1024$ bases. Test image sets are downloaded from public databases [52]–[55], and most of the image sets are landmarks and part of them are shown in Fig. 7.

Fig. 8 demonstrates the Rate-Distortion (RD) curves of the proposed method comparing with MP [56], RLS [32] as well as popular image compression standards JPEG and JPEG2000. The algorithm details of MP and RLS are given in Table III. For MP, the OMP_L sparse coding and ODL [31] dictionary learning algorithms are performed. RLS employs the RDOMP sparse coding and RLS [32] dictionary learning. In the pro-

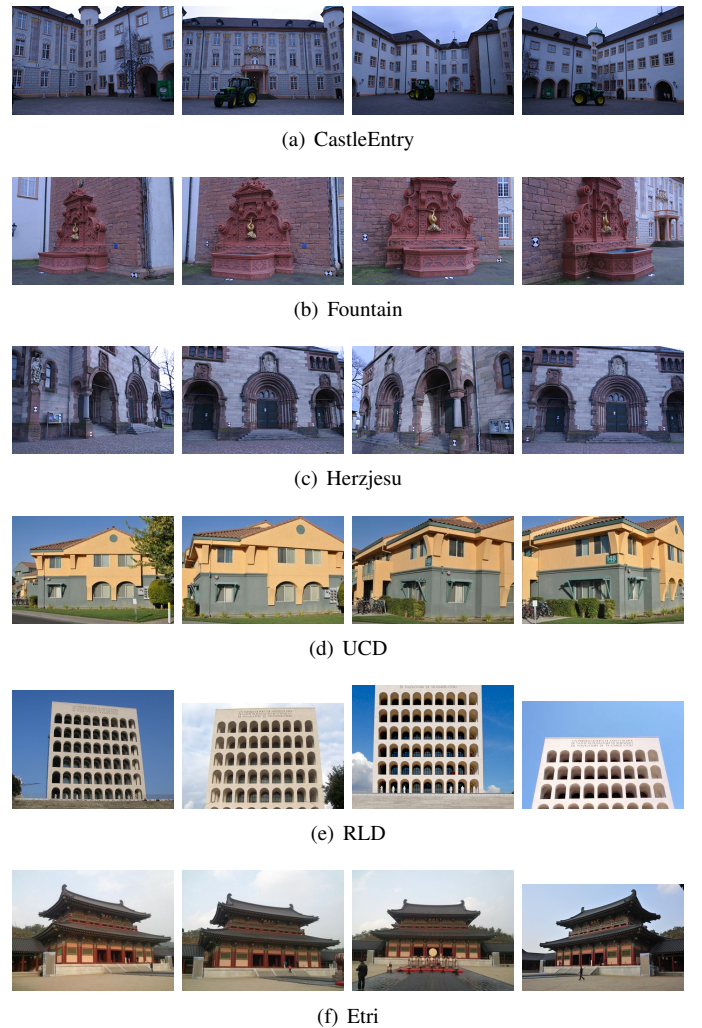


Fig. 7. Examples of the image sets [52]–[55].

TABLE III
ALGORITHMS USED IN MP, RLS AND THE PROPOSED IN IMAGE SET
COMPRESSION COMPARISONS.

Algorithms	Dictionary Learning	Sparse Decomposition
MP	ODL [31]	OMP_L
RLS	RLS [32]	RDOMP
Proposed	GVCSR	GVCSR

posed method, the GVCSR based sparse coding and GVCSR based dictionary learning are consistently utilized. It should also be noted that the rate and distortion are averaged across all the test images except the training ones in a set. From the results, we can observe that the proposed GVCSR model can obtain superior RD performance over MP and RLS. This may be originated from the rate-distortion joint optimization during both dictionary learning and sparse coding stages, which is critical to the final RD performance. Furthermore, coding gains against JPEG and JPEG2000 are also encouraging via the proposed scheme, indicating great potential of using the GVCSR method for practical compression applications.

V. CONCLUSION

In this work, we present a novel Globally Variance-Constrained Sparse Representation (GVCSR) for rate-distortion joint optimization. To achieve this goal, a variance-constraint term that can accurately predict the coding rate of the sparse coefficients is introduced into the optimization process. Subsequently, we propose to use the Alternating Direction Method of Multipliers (ADMM) to effectively solve this model in a scientifically sound way. In this manner, the rate-distortion jointly optimized sparse representation can be achieved, leading to higher compression efficiency. Furthermore, experimental results have shown that the GVCSR model can achieve better RD performance comparing with the state-of-the-art sparse coding and dictionary learning algorithms. We further demonstrate the effectiveness of the proposed method in image set compression, showing significant performance improvements over popular image compression algorithms.

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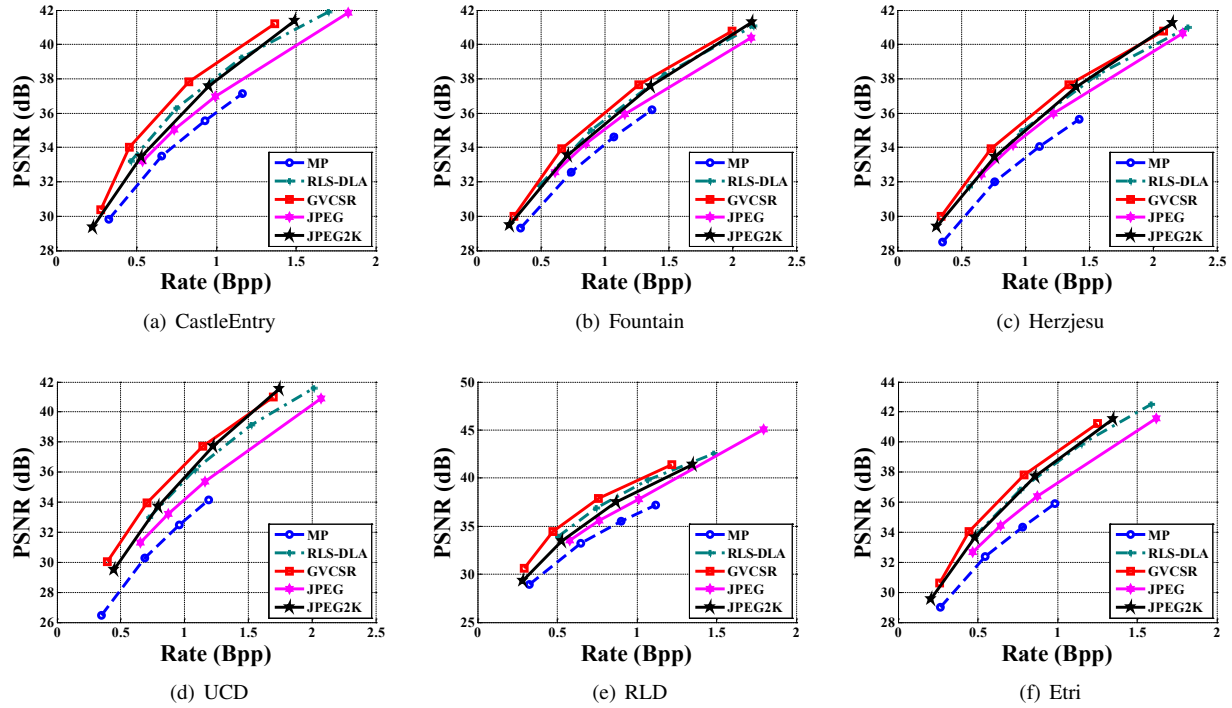


Fig. 8. Image set compression: RD performance comparisons with MP [56], RLS [32] as well as state-of-the-art image compression standards JPEG and JPEG2000.

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